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A GENERALIZING KALDOR NEO-PASINETTI MODEL WITH POLITICAL ORIENTATION AND CONSIDERING AN OPEN ECONOMY

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A Dissertation in Economics presented to the Master in Regional Economic Program at State University of Londrina as a partial fulfilment of the requirements for this degree.

Supervisor: Professor Dr. Renato Nozaki Sugahara (State University of Londrina)

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Londrina, May 30, 2019

In memory of my grandfather

Carlito Araujo

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RESUMO

A presente dissertação trata das atividades governamentais e do mercado financeiro no modelo Kaldor neo-Pasinetti. No começo, para consolidar o conhecimento do leitor com respeito ao tema é apresentado uma linha histórica com respeito as atividades governamentais nesta teoria de crescimento econômico de longo-prazo. É muito claro que no passado, todas as extensões deste modelo consideraram esta classe exógena em ambos os casos, política fiscal e monetária, entretanto, a natureza essencial da "Equação de Cambridge" é mantida. O modelo de Kaldor (1966) de crescimento e distribuição de renda não considerou governo no seu artigo, mas introduziu a ideia de mercado financeiro. Para gerar uma nova extensão, Charles (2007) publicou um artigo sobre as implicações negativas na distribuição de renda quando o governo expande o consumo em favor das famílias por meio de políticas orientadas. Assim, ele comete alguns erros matemáticos, que são corrigidos na sessão 2 deste trabalho. Depois disso, é mostrado aqui, quais são as implicações de políticas econômicas orientadas na distribuição de renda para ambas as classes (firmas e trabalhadores). Deste modo, é formulada uma nova extensão provando que as escolhas políticas, para ambos os casos, com e sem economia aberta, não afetam a natureza essencial do resultado da dinâmica de equilíbrio Kaldor neo-Pasinetti e da "Equação de Cambridge". Conexões com a teoria neo-Kaleckiana ou pós-Kaleckiana (wage-leg and export-led growth) são estabelecidas. Em seguida, é aplicado o Teorema de Olech que garante o equilíbrio estável em ambos os casos.

Palavras-Chave: Distribuição. Crescimento. Governo. Economia aberta. Estabilidade.

JEL: D30, O40, P16, F41, C62

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ABSTRACT

The present dissertation is concerned about the government activities and financial market in the Kaldor neo-Pasinetti model. In the beginning, to consolidate the knowledge of the reader to understand the field, it is presented a historical review about the government activities in long-run economic growth theory. It is clear that in the past, all the extensions of this kind of model consider this class exogenously in both cases, fiscal and monetary policy, however, the essential nature of the "Cambridge Equation" is maintained. Kaldor (1966) model of growth and income distribution, did not consider government in his paper, but was introduced some ideas of financial market. To develop an extension, Charles (2007) published an article about the negatively implications in the income distribution, when the government expand the consumption in favour to households from Political Orientations. Thus, he commit some mathematical mistakes, which is corrected in the section 2 of this work. After that, it is show, how is the implications of economy policy orientation in the income distribution considering incentives to both classes (firms and workers). That way, it is formulate a new extension proving that the political choice, in both case, with and without an Open Economy, does not affect the essential nature of the Kaldor neo-Pasinetti dynamic equilibrium results and the "Cambridge Equation". Connections with the neo-Kaleckians and post-Kaleckians theory (wage-led and export-led growth) are considered. Next it is applied the Olech's Theorem to guarantee that the equilibrium is stable in both cases.

Key words: Distribution. Growth. Government. Open economy. Stability.

JEL: D30, O40, P16, F41, C62

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NOTATIONS

α speed adjustment of government policy
β percentage destination of the Political Orientation to increase consumption
β^* equilibrium β
ρ implicit inflation tax
A total assets amount
A_c capitalists assets amount
A_w workers assets amount
AI aggregate investment
b marginal propensity to saving by workers from the income transferred by government
B total bonds
B_c capitalists bonds
B_w workers bonds
c marginal propensity to consume of the capital gain/loses
c_c propensity to consume of the capitalists
c_w workers propensity to consume
$\bar{c}$ consumption increased by the Political Orientation in capital terms
C general consumption
$\bar{C}$ general consumption increased by the Political Orientation
D_p net demand for placements
E Excess Demand
F financial international market
g_n natural growth rate
g_M money growth rate
g_Y income growth rate
G capital gains/loses
G_g government expenditures
g_i real value from consumption increased by Political Orientation in capital terms
G_i real value from consumption increased by Political Orientation
i nominal interest rate
I domestic investment
J Jacobian Matrix
$|J|$ Determinant of the Jacobian Matrix
K capital stock
K_c capital stock owned by capitalists
K_w capital stock owned by workers
M import
M_m money stock
N share of the firm in the financial market
p price level
$\dot{p}$ rate of finflation
P profit
P_c profit earn by capitalists
P_g profit earn by government
P_w profit earn by workers
$\bar{P}$ profit increased by the Political Orientation
r profit rate

R total amount remunerate by bonds
$\bar{r}$ profit increased by the Political Orientation in capital terms
R_c capitalists amount remunerate by bonds
R_g government payments from bonds
R_w workers amount remunerate by bonds
S total saving function
s_c capitalists saving rate
s'_c capitalist saving rate with government
S_c capitalist savings
s_f marginal propensity to save of the firms
S_f firms savings
s_g marginal propensity to saving of the government
S_g government saving
S_p supply of new securities issued by the corporations
s_w marginal propensity to save of the workers
s'_{wc} workers saving rate from profits and with government
s'_{ww} workers saving rate from wages and with government
S_w workers saving
t time
t_i indirect taxes
T amount tax
t_p marginal tribute to the profit
$T_r(J)$ trace of the Jacobian matrix
t_w marginal tribute to the wages
V speed of the money
v technology
v_r valuation ratio of the share in financial markets
W wages amount
X export
x share of the investment financiered by the existence of the financial market
Y income
y level of real output
z international amount of the international share in the economy in capital terms
z_c ownership share of the capitalists
Z amount of the international share in the economy

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1. INTRODUCTION

The theory of long-run growth macroeconomic analysis starts when Harrod (1939) and Domar (1947) present the “razor wire” problem, which implicates that the growth of the economy could be sustainable if the natural growth ratio is equal to the warranted growth ratio. By analysing this problem, Solow (1956) and Kaldor (1956) search for an alternative solution. The latter creates a theory of growth based on the side of income distribution. This effort shows us that the growth rate will be given by the multiplication between the propensity to save and the profit rate of the economy. Such result was named as “Cambridge Equation” and all extensions from this theorem have to return to the original result if the modifications are not considered.

Studying the kaldorian approach, Pasinetti (1962) divide the economy in two classes, workers and capitalists, saying that Kaldor (1956) committed a “logical sleep” when he did not consider the division of classes. Pasinetti proves that the “Cambridge Equation” is not given by de propensity to save of the economy, but only from the propensity to save of the capitalists. This indicates how much the economy will grow.

From this perspective, was created a line of think introducing in this kind of models the presence of the government. The first approach elaborated about this, was made by Steedman (1972), which consider just direct taxes in his extension and using this as a income transfer to workers. From this extension, researchers as Dalziel, Palley, Denicólo and Matteuzzi and others expand the model with different circumstances with fiscal and monetary policy, showing that the essential nature of the “Cambridge Theorem” is maintained in all the analysed cases.

Using this classes division, but considering the capitalists as part of the firms, Kaldor (1966) developed the “Kaldor neo-Pasinetti Theorem”. From this extension, is showed that how the existence of the financial system implicates on the income distribution. The Theorem is divided in two sides, the first side indicates a traditional profit rate and the second a valuation ratio of the firms in the financial market. The significant point of this theorem is that the profit rate affects negatively the valuation ratio, and the existence of financial assets leads to a reduction of the profit rate.

Concerned with to how the valuation ratio and the profit rate behave in Kaldor neo-Pasinetti model, Araújo (1995) reproduce graphically the theory and design the model as an IS-LM alternative. Another extension is presented by Panico (1997), which introduce the concept of capital gains to analyse how the existence of firms can impact the level of income and the implication on the income distribution with government activities. It is interesting to note that all the Political Orientations of the above extensions were assumed to be exogenous. Trying to endogenize these assumptions, Charles (2007) introduces the concept of investment increased by government expenditures involving two favouring choices. The first is to increase the consumption (households) and the second to expand the profit of the economy (firms).

The relevance of the Kaldor-Pasinetti theory persist in our days, Romero (2019) combine the kaldorian perspective with Schumpeter to present a new cumulative growth model. Another example is the empirical analysis to the case of the "Cambridge Equation". Pacheco-Lópes and Thirlwall (2014) verify the association between manufacturing output growth, export growth as well as between export growth and GDP growth in 89 developing countries. Bernardo, Stockhammer and Martínez (2016) shows the reinterpretation of the Tobin's q from the Kaldor neo-Pasinetti perspective. George (2018) made a positive analysis of the modern capitalism from the Pasinetti perspective to consider the implication of investors decisions in a long-run perspective.

The present dissertation has the focus in kaldorian perspective to analyse the interaction between government (especially introducing the Political Orientation to the model), financial market and Open Economy, intending to show the importance of these sectors to this kind of theory. Coming with a more realistic context and leading to a closer situation to the real world. This work is divided in four sections; the first one is this introduction which presents one trajectory of the Kaldor-Pasinetti model and some important extensions. The second, synthesizes the history of this theory and contextualizes the literature. In the same section we show the evolution of the "Kaldor neo-Pasinetti model" and their extensions. Also corrects the formal version of the model published by Charles (2007), since it contained a number of mistakes.

In the third section we extend the "Kaldor neo-Pasinetti model" showing the effective expansion of the consumption and/or profit by the government and the stability condition of this extension. This improved alternative has as consequence interesting results about the proper incentives of the Political Orientations to households. This leads to a positive relation with income distribution. In the two last subsection of this parte we extend our model to the case of an Open Economy showing the impact of this new configuration, as the same way in subsection 3.1 we use the Olech's Theorem for the stability condition as presented by Garcia (1972). The final section contains the concluding remarks.

2. THE EVOLUATION OF THE KALDOR-PASINETTI MODELS WITH GOVERNMENT IMPLICATIONS: THEOERICAL FUNDAMENTATION

In the 1940's, the macroeconomics thinking starts to focus in growth economic models, which is focused on the long-run perspective. Harrod (1939) and Domar (1947) concluded in their respective articles that an economy can only guarantee a permanent growth, if the natural growth rate and warranted growth rate is equal and this was named as "razor's edge problem". The objective of this part is to show how was developed one of the models and extensions which demonstrate the solution to this "problem", considering income distribution.

2.1. INTRODUCTION OF THE GOVERNMENT ACTIVITIES IN KALDOR-PASINETTI MODELS

Kaldor (1956) obtained a solution to the "razor's edge problem" by endogenizing the saving rate of the economy. This model is known as "Cambridge Equation", where the natural growth rate is explained by a multiplication between the profit rate and saving rate. Unfortunately, this author did not distinguish classes in his model. Pasinetti (1962) considered these assumptions and show that the "Cambridge Equation" is actually impacted only by capitalists' saving rate and determined the essential nature of the "Cambridge Theorem".

Meade and Hahn (1963) criticize this new extension and elaborate the "Dual Theorem". Assuming that government activities and the workers saving rate is bigger than the rate of investment, hence only workers will own the capital and the capitalism will end. In response to they, Steedman (1972) expands the model with government and shows that this "new" class does not affect the essential nature do the Pasinetti Theorem and increases the profit rate of the economy thus benefiting both classes.

This Kaldor-Pasinetti extension shows that worker's and capitalist's income is direct affected by their taxations, which constitute the government revenue. However, part of this amount is transferred to workers (bG_g). This concept was developed by Steedman (1972). It is important to stress that this extension did not consider unbalanced government budget and only considered direct tax to wages and profits. The capitalists savings are determined by profits while workers swing come from wages, profits and income transfer by government activities:

$$S_w = \frac{dK_w}{dt} = s_w[(1 - t_w)W + (1 - t_p)rK_w + bG_g] \quad (1)$$

$$S_c = \frac{dK_c}{dt} = s_c(1 - t_p)rK_c \quad (2)$$

Considering: $0 \leq t_p < t_w < 1$, $K_w \geq 0$, $0 \leq b \leq 1$, $K_c \geq 0$ and $0 < s_w < s_c \leq 1$.

We consider in this dissertation that the profit taxation rate has to be inferior to the wages taxation rate. Furthermore, the capitalist (firms) saving rate needs to be bigger than the workers saving rate. The equation (1) shows that is workers income increased by government activities and both equations (1 and 2) shows the impact of the taxations. From (2) it is easy to obtain with some mathematical manipulations, the Steedman extension. Thus:

$$r = \frac{g_n}{s_c(1-t_p)} \quad (3)$$

The equation (3) shows us that considering government in this model, does not imply in the “Dual Problem” and, more important, the essential nature of the “Cambridge Equation” is maintained. Steedman was criticized by Fleck and Domenghino (1987) to have not consider an “Open Economy”. To refute the idea presented by the last authors, Dalziel (1989) extend the model with international trade.

He defines two different assumptions of Steedman. The first one is the determination of the government saving function and this can be equal or different than zero and the second is consider a liquid exports in the model, which can be positive (surplus), zero or negative (deficit). These assumptions shows a new equilibrium, which is:

$$s_w(W + P_w) + s_cP_c + s_g(P_g + T) = I + NX \quad (4)$$

It is interesting to note that considering government saving, this class is allowed to own capital (investment), that way, as was showed in (4) where is explicit that government earn profit. With these assumptions, Dalziel developed the profit rate and the profit share considering an Open Economy. Thus:

$$\frac{P}{K} = \frac{I+NX}{s_c K} \quad (5)$$

$$\frac{P}{Y} = \frac{I+NX}{s_c Y} \quad (6)$$

If is consider $NX = 0$ we return to the Passinetti Theorem. From (5) and (6) Dalziel conclude that the government does not affect the essential nature of the "Cambridge Equation". However, in the case of a positive liquid exports, both equations increase and this will reflect in all classes. Since capitalists, workers and government earn profit.

Denicólo and Matteuzzi (1990) wrote a paper with the assumption of unbalanced budget, which was not considered by the authors above and this explain the behaviour of bonds transactions. These assets are remunerated by interest rate, which have to be equal to the profit rate by no-arbitrage condition. From this, they presented two new functions, which show the total assets of capitalists and workers:

$$A_c = K_c + B_c \quad (7)$$

$$A_w = K_w + B_w \quad (8)$$

Being $B_c \geq 0$ and $B_w \geq 0$. These assumptions affect the capitalists and workers saving functions:

$$S_w = s_w(1 - t_w)(W + P_w) + s_w i B_w \quad (9)$$

$$S_c = s_c(1 - t_p)P_c + s_c i B_c \quad (10)$$

From equation (10) the authors obtain the extension of the "Cambridge Equation" considering government unbalanced budget. Thus:

$$r = \frac{(1-t_p)P_c + iB_c}{K_c + B_c} \quad (11)$$

Where $B_c = 0$, $\frac{P}{K} = \frac{P_c}{K_c} = \frac{P_w}{K_w} = r$ and since the natural growth rate in Pasinetti is determined by $s_c r = g_n$, we have with some mathematical manipulations in equation (11) the

Steedman extension, concluding that the essential nature of the Pasinetti Theorem is maintained. The bonds propose, was also presented by Dalziel (1991), when he consider a small close economy, showing how of government titles affect the national income. Thus:

$$Y = [(1 - t_w)W + (P_w + R_w)(1 - t_p)] + (P_c + R_c)(1 - t_p) + (1 - t_p)P_g + T - R_g \qquad (12)$$

Being $R_w = rB_w \geq 0$, $R_c = rB_c \geq 0$, $R_g = -rB \leq 0$, $R_g = R_w + R_c$ and $B = B_w + B_c$. We have that, all the emitted government bonds, has to be remunerated by a interest rate equal to the profit rate in private sector. Consequently, the savings rates are equal as presented by Pasinetti (1989a). Thus:

$$s'_{ww} = s_w(1 - t_w) + s_g \propto [t_w + (1 - t_p)(s_w t_i + (1 - s_w)t_p] \qquad (13)$$

$$s'_{wc} = s_w(1 - t_p) + s_g \propto [t_w + (1 - t_p)(s_w t_i + (1 - s_w)t_p] \qquad (14)$$

$$s'_c = s_c(1 - t_p) + s_g \propto [t_p + t_i(1 - s_c)(1 - t_p)] \qquad (15)$$

$$S = s'_{ww}W + s'_{wc}P_w + s'_c P_c \qquad (16)$$

Being $\propto = [1 - t_i(1 - s_g)]^{-1}$ and $0 \leq t_i \leq 1$. Pasinetti (1989a) and Pasinetti (1989b) concluded that, if the government saving rate with unbalanced budget grows, the profit rate will be smaller, as is presented in the function below:

$$r = \frac{g_n}{s'_c} \qquad (17)$$

From (17), we conclude that the "Cambridge Equation" is affected by the government saving function and it is advised to this class not to keep large investments in this case. All of these extensions, show the fiscal side of the economy. However, it was not considered the monetary side in the models. The next subsection of this part is based on Dalziel (1991) and Palley (1997), where is considered a monetary police and is presented the answer to the following question: what is the implication of these models on the income distribution?

2.2. CAMBRIDGE EQUATION WITH MONETARY POLICY

The first kaldorian extension with monetary policy, was presented by Dalziel (1991). The author developed a version, which consider that the government activities are financed, in the steady-state, with money instead of bonds. However, if we assume $R_w = R_c = 0$ in the equation (12) and consider $\alpha > 0$ and $s_g < 0$ then from (13), (14) and (15) the follow can be verified: $s'_{ww} < s_w(1 - t_w)$; $s'_{wc} < s_w(1 - t_p)$; $s'_c < s_c(1 - t_p)$.

Pasinetti (1989a) intended to maintain the consistence of the "Cambridge Theorem" suggest an implicit inflation tax, which has to be equal to the taxations, obtaining the result $s'_c \geq s_c$. This rate of inflation shows the reduction of the workers and capitalists savings rate considering government savings in the economy:

$$(1 + \rho) = \frac{S_w + S_c}{S} = \frac{S_w + S_c}{S_w + S_c + S_g} \tag{18}$$

Assuming that, in steady-state equilibrium, the total share of the capital stock held by capitalists is:

$$\frac{K_c}{K} = \frac{s_c(1 - t_p)P_c(1 + \rho)}{S} \tag{19}$$

In the long-run, we have: $S = I$ and $\frac{P_c}{K_c} = \frac{P_w}{K_w} = \frac{P}{K} = r$. From these and with some mathematical manipulations, we can obtain the 'Cambridge Equation" extension with monetary policy:

$$r = \frac{(1 + \rho)g_n}{(1 - t_p)s_c} \tag{20}$$

The equation (20) did not affect the essential nature of the Pasinetti Theorem and considering inflation in the model, we have a bigger rate of profit comparing to the extensions without inflation.

Palley (1997) to extend the model, considering money stock as an assumption, which is divided from capitalists and workers. This propose show us that the investments can be derived from government debts. He also defines the rate of inflation, which is represented by the difference between money growth rate and income growth rate[1]:

$$\frac{\dot{p}}{p} = g_M - g_y \tag{21}$$

This new equation impacts on the structure of the "Cambridge Equation" and the profit share. Thus:

$$\frac{P}{K} = r = \frac{I}{s_c[1-t_p]K} + \frac{\dot{p}c_c M_m}{s_c[1-t_p]z_c K} \tag{22}$$

$$\frac{P}{Y} = \frac{I}{s_c[1-t_p]Y} + \frac{\dot{p}c_c M_m}{s_c[1-t_p]z_c Y} \tag{23}$$

Being $0 \leq c_c \leq 1$, $0 \leq z_c \leq 1$ and $M_m \geq 0$ and considering the rate of inflation equal to zero, the equation (22) and (23) return to the results presented by Steedman (1972). Proving that the essential nature of the "Cambridge Equation" is maintained. These proposes and considering flexible prices, Palley (1997) shows that the liquid savings amount derived from bonds affect the structure of the model. Thus:

$$\frac{P}{K} = \frac{I}{s_c[1-t_p](K+B)} - \frac{\dot{p}\{Bz_c s_c[1-t_p]-c_c M_m\}}{z_c s_c[1-t_p](K+B)} \tag{24}$$

$$\frac{P}{Y} = \frac{IK}{s_c[1-t_p](K+B)Y} - \frac{\dot{p}\{Bz_c s_c[1-t_p]-c_c M_m\}K}{z_c s_c[1-t_p](K+B)Y} \tag{25}$$

However, it is also maintained the essential nature of the Theorem and it is possible to return to the Steedman extension considering the variation inflation in respect to time and bonds

[1] In steady-state, the price level is determinate by $py = M_m V$, considering V constant, and deriving with respect to time, we have (21), as we can see in Palley (1997).

equal to zero. All of these models presented above did not consider financial market or Political Orientation. Kaldor (1966) starts a whole new extensions considering equities as an asset in the model and Charles (2007) published a paper showing the Political Orientation to firms or households. The next subsection will present this.

2.3. REVIEW OF THE "KALDOR NEO-PASINETTI MODEL" AND EXTENSIONS

Kaldor (1966) assume that there is only two classes, firms and workers. The first agent is owned by capitalists and workers[2] (households) and the profits is divided between then. The last agent will also be remunerated with wages earned from his\her jobs. The sum of these two remunerations is equal to the national income. The savings functions are similar to the one used by Pasinetti (1962) when he made the distinction between workers and of capitalists. Following Kaldor (1966) and Charles (2007) assumptions[3]:

$$Y = W + P \tag{i}$$

$$S = S_w + S_f \tag{ii}$$

$$S_w = s_w W - cG \tag{iii}$$

$$S_f = s_f P \tag{iv}$$

$$xI = s_w W - cG \tag{v}$$

Been $0 \leq s_w < s_f \leq 1$. The difference from the equations above and Kaldor (1956) is that he and Charles (2007) consider that the total saving is composed by the one of workers and that of firms. The new part of the investment function derived from the financial market is given by (v). An interesting property of this equation is to show that the existence of financial market will be direct affected from workers savings[4].

Following the assumptions by Kaldor (1966), Araújo (1992), Charles (2007) and Lavoie (2014), the equilibrium between investments and saving to maintain the full employment has to be written as in (vi) and (vii):[5]

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[2] It is possible to consider that not all the workers own part of the firms, but those who have earn profits. This concept is maintained in the rest of this dissertation.

[3] The assumptions hereafter is defined by Roman numbers.

[4] We are considering in this dissertation that households only saving from wages and firms, which earn profit, share this income between households and the saving is part is designated to investments, as in Kaldor (1966).

[5] All the investments functions in this dissertation are based on the Keynes perspective, where this variable is exogenous and the distribution is determinate from the saving function, as we can see in Bertola (2000).

$$\mathrm{I} = \mathrm{s_f P} + \mathrm{xI} \tag{vi}$$

$$\mathrm{I} = \mathrm{s_f P} + \mathrm{s_w W} - \mathrm{cG}, \text{ been } 0 \leq \mathrm{c} \leq 1 \tag{vii}$$

The capital gains in (v) and (vii) is determined from financial market equilibrium, where the valuation ratio is equal to the amount value of equities divided by the total capital stock. That is:

$$\mathrm{v_r} = \frac{\mathrm{pN}}{\mathrm{K}} \tag{viii}$$

Deriving (viii) with respect to time and applying some algebraic manipulations, we have the capital gains function. This results can be either positive (capital gain), zero or negative (capital loses). Than:

$$\mathrm{G} = (\mathrm{v_r} - \mathrm{x})\mathrm{I} \tag{ix}$$

Substituting (ix) in (vii), Kaldor (1966), Araujo (1995), Charles (2007) obtained the main results like as presented in Lavoie (2014). First of all, they find the profit rate "Cambridge Equation" (equation 26), showing that the existence of financial market will make a decreasing impact on the profit rate. The other result presents the valuation ratio (27), which indicates the signal of capital gains. The rate of investment by financial market is impacted positive in the valuation ratio:

$$\mathrm{r} = \frac{(1-\mathrm{x})\mathrm{g}_n}{\mathrm{s_f}} \tag{26}$$

$$\mathrm{v_r} = \frac{1}{\mathrm{c}}\left[\frac{\mathrm{s_w}}{\mathrm{g}_n \mathrm{v}} - \frac{\mathrm{s_w}}{\mathrm{s_f}}(1-\mathrm{x}) - \mathrm{x}(1-\mathrm{c})\right] \tag{27}$$

Following assumptions (i) to (ix), Panico (1997) introduced the government expenditures (x) which increase the investment function (vii). He assumed that the government budget is balanced as expressed by (x) and (xi):

$$\mathrm{G_e} = \mathrm{T} \tag{x}$$

$$\mathrm{T} = \mathrm{t_w W} + \mathrm{t_p P}, \text{ been } 0 \leq \mathrm{t_p} < \mathrm{t_w} \leq 1 \tag{xi}$$

Assuming the existence of government and the respective direct taxation, we obtain the right landside of the equations (xii) and (xiii) which, in equilibrium, show that investment is equal to total amount of saving, as expressed by (xiv):

$$I + G_e = s_f(1 - t_p)P + s_w(1 - t_w)W - cG \tag{xii}$$

$$xI = s_w(1 - t_w)W - cG \tag{xiii}$$

$$I + G_e = s_f(1 - t_p)P + xI \tag{xiv}$$

After some mathematical manipulations, Panico (1997) shows new extensions of Kaldor (1966) approach now with government activities, as presented in (28) and (29). The interesting part of these results is concerned with the existence of the government expenditures affecting positively the profit rate and negatively the valuation ratio as we can see in the equations below (we just have to make the partial derivate of r and v_r with the respect to g_e):

$$r = \frac{(1-x)g_n + g_e}{s_f(1-t_p)} \tag{28}$$

$$v_r = \frac{1}{c}\left\{\frac{s_w}{vg_n}(1 - t_w) - \frac{s_w}{g_n}(1 - t_w)\left[\frac{(1-x)g_n + g_e}{s_f(1-t_p)}\right] - x(1 - c)\right\} \tag{29}$$

Note that $g_e = \frac{G_e}{K}$. However, the government expenditures decisions are exogenous. Wondering about this, Charles (2007) elaborated a kaldorian extension model with Political Orientation which deserves especial attention. The section V of his article different ides the government orientation in two ways. The first was to increase the consumption in favour to households and the second to increase profit favouring to firms. Being $0 \leq \alpha \leq 1$, we have:

$$G_e = \alpha(\bar{C} - C),\ \bar{C} > C, \text{ been } 0 \leq \alpha \leq 1 \tag{xv}$$

$$G_e = \alpha(\bar{P} - P),\ \bar{P} > P \tag{xvi}$$

The equation of the consumption (xvii) incorporates the taxation:

$$C = c_w(1 - t_w)W + (1 - s_f)(1 - t_p)P + cG \tag{xvii}$$

Mathematical manipulations in the Appendix 1 indicate that Charles (2007) committed some mathematical mistakes. Our first contribution corrects his results in relation to the consumption incentives. His model disappeared with the difference between valuation ratio and the share of investments financed by the financial market $(v_r - x)$. The actual results are given by (30) and (31):

$$r = \frac{g_n[1-\alpha c(v_r - x)] + \alpha\bar{c} - \frac{\propto}{v}(1-t_w)}{(1-t_p)[\alpha(1-s_c)-s_c] - \propto(1-t_w)} \tag{30}$$

$$v_r = \frac{x g_n \theta(1-c) + s_w(1-t_w)\left\{\frac{\theta}{v} g_n + \alpha\left[\frac{(1-t_w)}{v} - \bar{c} - g_n c x\right]\right\}}{c g_n \theta - s_w(1-t_w) g_n \alpha c} \tag{31}$$

where $\bar{c} = \frac{\bar{C}}{K}$ and $\theta = (1-t_p)[\propto (1-s_c) - s_c] - \alpha(1-t_w)$.

With the same manipulations, but considering (xvi) we have the equations favouring profits (firms), as expressed by (32) and (33). These results are expressed correctly by Charles (2007).

$$r = \frac{\alpha\bar{r} + g_n(1-x)}{s_c(1-t_p)+\alpha} \tag{32}$$

$$v_r = \frac{1}{c g_n}\left[\frac{s_w}{v}(1-t_w) - g_n(1-c)x - s_w \frac{\alpha\bar{r} + g_n(1-x)}{s_c(1-t_p)+\alpha}\right] \tag{33}$$

Equations (30) and (31) show us that the government expenditure to incentive the households will affect the income distribution negatively. In addition, the equation (32) and (33) indicate that government expenditures tend to increase profit and the profit ratio will be bigger, favouring the firms.

A number of extensions, analyse cases and empirical applications about Kaldor neo-Pasinetti approach are recently published by Park (2002), Ryoo (2016) and Ryoo (2018). Thinking about the incentives and fiscal policy we realized that some issues were not considered in the extensions above. One of our main contribution is to present how much bigger it will be $\bar{C} > C$. From this assumption we develop a new extension of the "Kaldor neo-Pasinetti Theorem" with proper Political Orientation. We formulate this new approach in section 3 of this paper.

3. EXTENSION OF THE KALDOR NEO-PASINETTI MODEL WITH PROPER POLITICAL ORIENTATION

This part of the dissertation focus on a generalization of the “Kaldor neo-Pasinetti Theorem”, concerning governmental economic Political Orientation favouring consumption (benefit to households) and/or profit (benefit to firms), from government preferences determined by political power. We will different from Charles (2007) in two ways: the first is that we are considering that the government can incentive simultaneously both classes in the model and what happens in our new extension when policymakers decide to increase consumption. This is defined here as the sum of the consumption of the economy and the incentive derived from the government expending (income transfer).

Assuming the assumptions below:

$$G_e = \beta\alpha(\bar{C} - C) + (1 - \beta)\alpha(\bar{P} - P) \qquad \text{(xviii)}$$

$$0 \leq \beta \leq 1 \qquad \text{(xix)}$$

$$\bar{P} > P \qquad \text{(xx)}$$

$$\bar{C} > C \qquad \text{(xxi)}$$

$$\bar{C} = C + G_i \qquad \text{(xxii)}$$

For each economy, we have a β determined, and the decision of how much it will be expended in incentive to consumption and/or profit is defined by $G_e = \max\{\alpha G_i , \alpha(\bar{P} - P)\}$. From these values the government will distribute the incentives using (xviii)[6]. If we consider $\beta = 1$, we have as a particular case, the Charles condition to incentive households and with $\beta = 0$ the incentive to firms. Note that different from Charles (2007) our expression (xxii) introduce the increased amount of consumption by the current policy favouring households (G_i). Following Kaldor (1966), Panico (1997) and Charles (2007) we have:

$$S = S_w + S_f = s_f(1 - t_p)P + s_w(1 - t_w)W - cG \qquad \text{(xxiii)}$$

[6] For an example, consider the following values: $\beta = \frac{1}{2}$, $\alpha = 1$, $G_i = 4$ and $\bar{P} - P = 2$. From these values we have $G_e = \max\{4 , 2\}$ and using (xviii), $G_e = 0{,}5 * 4 + 0{,}5 * 2 = 3$, which determinate 2 for incentives to consumption and 1 for incentives to profit.

$$T = t_w W + t_p P \qquad \text{(xxiv)}$$

Substituting (xxii) in (xviii):

$$G_e = \beta\alpha(G_i) + (1-\beta)\alpha(\overline{P} - P) \qquad (34)$$

Substituting (34) and (ix) in (xii), dividing such equation by K given that: $g_n = \frac{I}{K}$, $r = \frac{P}{K}$, $\bar{r} = \frac{\overline{P}}{K}$, $v = \frac{K}{Y}$ and $g_i = \frac{G_i}{K}$, it follows:

$$g_n + (1-\beta)\alpha\bar{r} - (1-\beta)\alpha r + \beta\alpha g_i = s_f\left(1-t_p\right)r + \frac{s_w}{v}(1-t_w) - s_w(1-t_w)r - c(v_r - x)g_n \qquad (35)$$

Isolating $\frac{s_w}{v}$ and rearranging the equation:

$$\frac{s_w}{v} = \frac{1}{(1-t_w)}\left[g_n + (1-\beta)\alpha\bar{r} - (1-\beta)\alpha r + \beta\alpha g_i - s_f\left(1-t_p\right)r + s_w(1-t_w)r + c(v_r - x)g_n\right] \qquad (36)$$

Take into account (xv), dividing the equation by K and isolating $\frac{s_w}{v}$, we obtain:

$$\frac{s_w}{v} = \frac{1}{(1-t_w)}[xg_n + s_w(1-t_w)r + c(v_r - x)g_n] \qquad (37)$$

Equating equations (36) and (37) and following steps (a) to (c) we, obtain r, expressed by equation (11):

$$\frac{1}{(1-t_w)}\left[g_n + (1-\beta)\alpha\bar{r} - (1-\beta)\alpha r + \beta\alpha g_i - s_f\left(1-t_p\right)r + s_w(1-t_w)r + c(v_r - x)g_n\right] = \frac{1}{(1-t_w)}[g_n + s_w(1-t_w)r + c(v_r - x)g_n] \qquad \text{(a)}$$

$$g_n + (1-\beta)\alpha\bar{r} - (1-\beta)\alpha r + \beta\alpha g_i - s_f(1-t_p)r = xg_n \quad \text{(b)}$$

$$g_n(1-x) + (1-\beta)\alpha\bar{r} + \beta\alpha g_i = r[(1-\beta)\alpha + s_f(1-t_p)] \quad \text{(c)}$$

$$r = \frac{g_n(1-x)+(1-\beta)\alpha\bar{r}+\beta\alpha g_i}{[(1-\beta)\alpha+s_f(1-t_p)]} \quad (38)$$

It is easy to see that the “Kaldor neo-Pasinetti Theorem” can be obtained by assuming that $\alpha = 0$. Furthermore, the extension of the “Cambridge Equation”, by Steedman (1972), is obtained if $\alpha = x = 0$. We get Pasinetti (1962) if $\alpha = x = t_p = 0$.

Relaxing the assumption of full capacity utilization, we take into account the implications of non-competitive markets and their imperfections[7]. To determine an investment function, as in models *à la* Kalecki, it is possible to construct a new version of our extension with wage-led and profit led growth view. Another accomplishment to consider, instead of to propose the incentives to consume, we determinate income transfers to workers and the analysis of how will behave the fluctuation of the income in short, medium[8] as well as to determinate the equilibrium in long-run term from Kalecki’s and Kaldor’s visions.A desirable extension, linking the idea of Political Orientation and Kaleckians views, could be supported by the following sentence in Lavoie and Stockhammer (2012, p. 1):

> Income distribution can be modified or influenced by appropriate government policies that act both on primary income distribution, for instance by reinforcing the bargaining power of labour unions or securing low real interest rates and on secondary income distribution, by modifying the tax code. (LAVOIE AND STOCKHAMMER, 2012, p. 1)

To deal with the interrelation between neo-kaleckian (post-Kalekian[9]) and kaldorian models, in the case of small close economy for both theories, Araujo and Teixeira (2015) as

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[7] The study more precisely the relation between capacity utilization and non-competitive markets, imperfections in the labour market (unemployment), we indicate Hein (2014, p. 241-271), which treat about the Kaleckians basic models.

[8] As was treated by Ros (2016), when he distinguish both short and medium term to determinate the impact of a wage-led growth theory in an small developing economy with two sectors.

[9] The difference between neo and post-Kaleckian are that the first one has as a precursors Dutt (1984) and Rowthorn (1981) works, which is known as wage-led theories and the second one start from Bhaduri and Marglin (1990) and Kurz (1990) works, as we can see in Hein (2017). Hein (2014) show that the difference between these models is the investment function, which leads the second group of authors to determinate wage-led and profit led growth models.

well as Araujo and Teixeira (2016) reinterpreted the "Cambridge Equation", which can be viewed as particular case of the Kalecki School, if we consider that the sensibility of the growth rate of investment been zero.

Substituting (38) in (37) we obtain:

$$\frac{s_w}{v} = \frac{1}{(1-t_w)}\left[xg_n + s_w(1-t_w)\left[\frac{g_n(1-x)+(1-\beta)\alpha\bar{r}+\beta\alpha g_i}{[(1-\beta)\alpha+s_f(1-t_p)]}\right] + c(v_r - x)g_n\right] \tag{39}$$

Isolate the v_r from (39):

$$v_r = \frac{1}{cg_n}\left\{\frac{s_w}{v}(1-t_w) - s_w(1-t_w)\left[\frac{g_n(1-x)+(1-\beta)\alpha\bar{r}+\beta\alpha g_i}{[(1-\beta)\alpha+s_f(1-t_p)]}\right] - x(1-c)g_n\right\} \tag{40}$$

In the same way that r was analysed from (38), we can obtain from (40) the Kaldor neo-Pasinetti original model if $\alpha = t_p = t_w = 0$. Both (38) and (40) has as a especial case, considering $\beta = 0$, the Charles extension with Political Orientation to firms[10]. These equations show a new general extension of the "Kaldor neo-Pasinetti Model". Something important from equations (38) and (40) are that they show a new general extension of the "Kaldor neo-Pasinetti Model"[11]. The present results refute the affirmation by Charles (2007), since his model does not hold if the government expenditure follows a specific budgetary policy favouring the households (increasing their consumption).

In Appendix 2 (a), we present one possible value to the equilibrium between the rate of profit and valuation ratio, of this economy. To see how r and v_r behave, we can apply the partial derivations: $\frac{\partial r}{\partial g_i} > 0$; $\frac{\partial r}{\partial \bar{r}} > 0$ and $\frac{\partial v_r}{\partial g_i} < 0$; $\frac{\partial v_r}{\partial \bar{r}} < 0$. These results show that if the government activities favour the consumption (households), it follows that the income distribution will not be affected negatively. Consequently the profit ratio will increase. Note that we have been dealing with a model for small closed economy. In order to generalize our approach, we extended it to the case of an Open Economy in section 3.2.

[10] Based on an erroneous assumption to increase consumption, determinate by Charles it is not possible to obtain from our equations (10) and (11) his results.
[11] We present a numerical simulation in the Appendix 2 (a).

3.1. STABILITY ANALYSIS OF THE MODEL WITHOUT AN OPEN ECONOMY

In this subsection we apply the Olech's Theorem[12] to analyse the stability of our extension.

From the equation (35) we obtain:

$$\frac{dr}{dt} = E(r, v_r) = \delta\left[g_n - \frac{s_w}{v}\right] = \delta\left\{g_n - \frac{s_w}{v}(1 - t_w) - \left[(1-\beta)\alpha + s_f\left(1 - t_p\right) - s_w(1 - t_w)\right]r + cg_n v_r - cxg_n + (1-\beta)\alpha\bar{r} + \beta\alpha g_i\right\}, \delta > 0 \quad (41)$$

Considering that the net demand for placements (xxv) is equal to the workers savings less the consumption from capital gains and the supply of new securities issues by the corporation (xxvi) as in Davidson (1968) and Araújo (1995), we have:

$$D_p = s_w(1 - t_w)W - cG \quad \text{(xxv)}$$

$$S_p = xI\left(\frac{K}{K}\right) = xg_nK \quad \text{(xxvi)}$$

Following the conventional IS-LM stability analysis, we postulate the equilibrium adjustment between r and v_r, represented by the excess demand function below:

$$\frac{dv_r}{dt} = E(r, v_r) = \varphi\left[\frac{D_p}{K} - \frac{S_p}{K}\right] = \varphi\left[\frac{s_w}{v}(1 - t_w) - s_w(1 - t_w)r - cg_n v_r + cxg_n - xg_n\right], \varphi > 0 \quad (42)$$

From (42) and (43) we can analyse the stability condition, considering the first term of the Taylor expansion. From this, we are allowed to determinate the matrix system:

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[12] The Theorem is presented in the Appendix 3.

$$\begin{bmatrix} \dot{r} \\ \dot{v}_r \end{bmatrix} = \begin{bmatrix} -\delta\left[(1-\beta)\alpha + s_f\left(1-t_p\right) - s_w(1-t_w)\right] & \delta c g_n \\ -\varphi s_w(1-t_w) & -\varphi c g_n \end{bmatrix} * \begin{bmatrix} r - r_0 \\ v_r - v_{r0} \end{bmatrix} \quad (43)$$

The matrix in the middle is the Jacobian Matrix:

$$J[E(r, v_r)] = \begin{bmatrix} -\delta\left[(1-\beta)\alpha + s_f\left(1-t_p\right) - s_w(1-t_w)\right] & \delta c g_n \\ -\varphi s_w(1-t_w) & -\varphi c g_n \end{bmatrix} \quad (44)$$

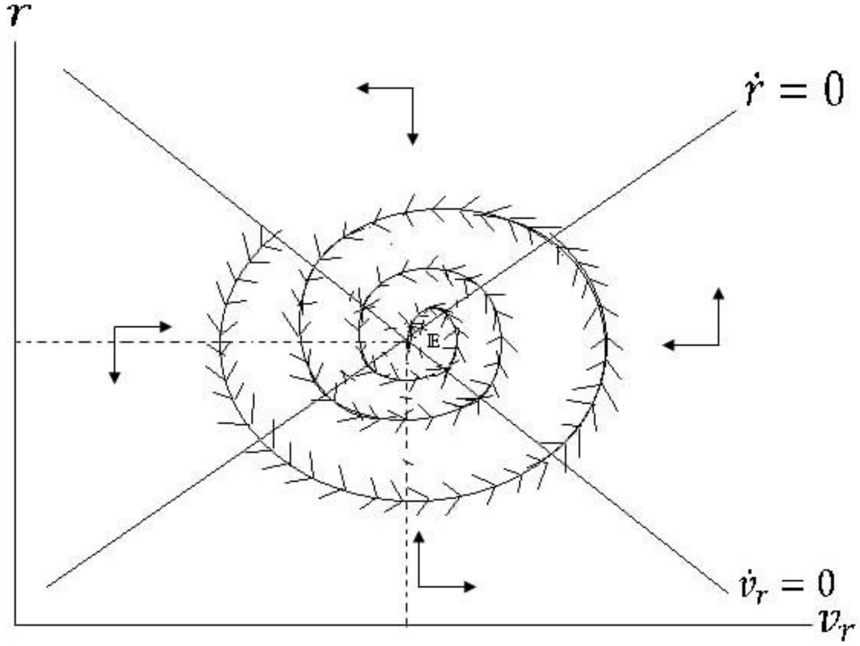
Applying the Olech's Theorem in (45), we have all the tools to analyse the stability in a Matrix 2x2. This is a necessary and sufficient condition, if the trace is negative and the determinant is positive, thus:

$$Tr(J) = -\delta\left[(1-\beta)\alpha + s_f\left(1-t_p\right) - s_w(1-t_w)\right] - \varphi c g_n < 0 \quad (45)$$

$$|J| = \delta\varphi c g_n\left[(1-\beta)\alpha + s_f\left(1-t_p\right)\right] > 0 \quad (46)$$

These results show us that with all the assumptions assumed above, the model is stable as required, *Quod Erat Demonstrandum*. Furthermore, we conclude that our extension satisfied the stability conditions. As we can see graphically in the Figure 1 below:

Figure 1: Dynamic Equilibrium without an Open Economy.



Source: Elaborated by the author.

Equalizing the equation (38) and (40), we can determinate the value of the equilibrium β to our extension. Thus:

$$\beta^* = \frac{\frac{s_w}{v}(1-t_w)[\alpha+s_f(1-t_p)]-x(1-c)g_n[\alpha+s_f(1-t_p)]-[g_n(1-x)+\alpha\bar{r}][cg_n+s_w(1-t_w)]}{(\alpha g_i-\alpha\bar{r})[cg_n+s_w(1-t_w)]+\frac{s_w}{v}(1-t_w)\alpha-x(1-c)g_n\alpha} \tag{47}$$

The expression determinate the β when $r = v_r$, as is presented in the Figure 1. With some numerical exercise (we advise to use the values expressed in the Appendix 2), it is possible to conclude that a modifying in the parameters α , s_f or t_p in the equilibrium, only affect the value of β^*. However, if we increase or decrease the value of s_w , v , t_w , g , x or c both values of β^* and the equilibrium will be altered.

3.2. AN EXTENSION OF THE KALDOR NEO-PASINETTI MODEL WITH POLITICAL ORIENTATION AND OPEN ECONOMY

This subsection will concentrate on the implications of an Open Economy in a "Kaldor neo-Pasinetti Model" with economic policy. We consider what happens if our extension considering the financial international market and how this can improve the income distribution. Such formalized concern came from Kaldor (1966). We are considering an income function presented by Metcalfe and Steedman (1979) and also used by Teixeira & Araújo (1997) when the latter introduced the idea of foreign bonds. Thus:

$$\mathrm{Y = W + P + F} \tag{xxvii}$$

Being, $\mathrm{F = iZ}$ and $\mathrm{i = r}$, the new assets are holding by firms and will be introduced in the general saving of an Open Economy impacting the aggregate investments. Let (xxviii) be the saving function and (xxix) the aggregate investment:

$$\mathrm{S = S_w + S_f = s_w(1 - t_w)W + s_f\left(1 - t_p\right)(P + F) - cG} \tag{xxviii}$$

$$\mathrm{AI = I + G_e + M - X + rZ} \tag{xxix}$$

In equilibrium we have:

$$\mathrm{I + G_e + X - M + rZ = S_w + S_f = s_w(1 - t_w)W + s_f\left(1 - t_p\right)(P + rZ) - cG} \tag{xxx}$$

From such three assumptions we can construct an extension of "Kaldor neo-Pasinetti Theorem", with Political Orientation and an Open Economy. After some mathematical manipulation, as in section 3. By substituting (34), (ix) and (xxvii) in (xxx), we obtain:

$$\mathrm{I + \beta\alpha(G_i) + (1 - \beta)\alpha(\overline{P} - P) + X - M + rZ = S_w + S_f = s_w(1 - t_w)W + s_f\left(1 - t_p\right)(P + rZ) - c(v_r - x)I} \tag{48}$$

That is, how the government can be allowed to choose the percentage designated to the Political Orientation. Dividing the equation (48) by K, considering: $\mathrm{\frac{Z}{K} = z}$ and $\mathrm{\frac{X-M}{K} = (g_n - i)z}$, isolating $\mathrm{\frac{s_w}{v}}$ we have:

$$\frac{s_w}{v} = \frac{1}{1-t_w}\left[g_n + (1-\beta)\alpha\bar{r} - (1-\beta)\alpha r + \beta\alpha g_i + g_n z - s_f\left(1-t_p\right)r - s_f\left(1-t_p\right)rz + s_w(1-t_w)r + s_w(1-t_w)rz + c(v_r - x)g_n\right] \quad (49)$$

Equalizing (49) with (37), after some algebraic manipulations we obtain the profit rate (49):

$$\frac{1}{1-t_w}\left[g_n + (1-\beta)\alpha\bar{r} - (1-\beta)\alpha r + \beta\alpha g_i + g_n z - s_f\left(1-t_p\right)r - s_f\left(1-t_p\right)rz + s_w(1-t_w)r + s_w(1-t_w)rz + c(v_r - x)g_n\right] = \frac{1}{(1-t_w)}[xg_n + s_w(1-t_w)r + c(v_r - x)g_n]$$

$$\therefore g_n + (1-\beta)\alpha\bar{r} - (1-\beta)\alpha r + \beta\alpha g_i + g_n z - r\left[s_f\left(1-t_p\right) + s_f\left(1-t_p\right)z - s_w(1-t_w)z\right] = xg_n$$

$$r = \frac{g_n(1-x) + (1-\beta)\alpha\bar{r} + \beta\alpha g_i + g_n z}{(1-\beta)\alpha + s_f\left(1-t_p\right)(1+z) - s_w(1-t_w)z} \quad (50)$$

The crucial point to notice is that equation (50) is an extension of the "Cambridge Equation" with Political Orientation and an Open Economy. This extension can be linked to the neo-Kaleckian theory, when the export-led growth model is considered, which was started by Blecker (1989) and Bhaduri and Marglin (1990). The first one, was concerned about the relationship between income distribution and international competitiveness, being extended by the seconds, introducing the real exchange rate to the model.

Hein (2014, Chapter 7) shows that the exchange rate is the cause of redistribution in this kind of models and he also presents the positive relationship between profit share and international competitiveness. These assumptions can be interesting, if we link to our model and analyse the consequences[13]. To introduce the financial sector, it is possible to follow the

[13] Bhaduri and Marglin (1990) and Hein (2014, p. 290) show that we have to consider a domestic and foreign capacity utilization to deal with the demand of import and export in the export-led growth models.

investment function presented by Arestis, González-Martinez and Dejuán (2016), when they analyse the relation between capital accumulation and financial market.

From that, as was done by Araujo and Lima (2007), it is possible to analyse the implications of the balanced-payments-constrained growth in different economic structures or structural changes. Following their proposal it is possible to construct a new Kaleckian extension to analyse some implications of our assumptions on Political Orientation and Open Economy from Financial Globalization[14].

If the reader has the intention to deepen in others kinds of methodologies treating about an Open Economy, we advise to read Romero and McCombie (2017), especially for the case considering Thirlwall's Law.

The Kaldor neo-Pasinetti model also present the valuation ratio. Substituting (50) in (37), after some mathematical procedure we obtain (51):

$$\frac{s_w}{v} = \frac{1}{(1-t_w)}\left\{xg_n + s_w(1-t_w)\left[\frac{g_n(1-x)+(1-\beta)\alpha\bar{r}+\beta\alpha g_i+g_n z}{(1-\beta)\alpha+s_f(1-t_p)(1+z)-s_w(1-t_w)z}\right] + c(v_r - x)g_n\right\}$$

$$\rightarrow cv_r = \frac{s_w}{v}(1-t_w) - s_w(1-t_w)\left[\frac{g_n(1-x)+(1-\beta)\alpha\bar{r}+\beta\alpha g_i+g_n z}{(1-\beta)\alpha+s_f(1-t_p)(1+z)-s_w(1-t_w)z}\right] - x(1-c)g_n$$

$$v_r = \frac{1}{cg_n}\left\{\frac{s_w}{v}(1-t_w) - s_w(1-t_w)\left[\frac{g_n(1-x)+(1-\beta)\alpha\bar{r}+\beta\alpha g_i+g_n z}{(1-\beta)\alpha+s_f(1-t_p)(1+z)-s_w(1-t_w)z}\right] - x(1-c)g_n\right\} \quad (51)$$

The equations (50) and (51) show that in the case of an Open Economy and Political Orientation we will have a positive result for the profit ratio[15], which has as a particular case, considering $z = 0$, our first extension presented above. On the other hand, like in Kaldor (1966), the implication of the profit ratio will be negative to the valuation ratio. The partial derivative of profit ratio in relation to the foreign equities is negative $\left(\frac{\partial r}{\partial z} < 0\right)$. However, to the valuation ratio it will be positive $\left(\frac{\partial v_r}{\partial z} > 0\right)$. Refuting the conclusion by Charles (2007) about the negatively implication of favouring households.

[14] It is essential to know that, one important difference between the Kaldorians perspectives and the neo-Kaleckians approaches on the capacity utilization is that in the first case is assumed full employment and in the second one is not. These imply in a great difference in studies about structural change and economic structures. We recommend to read Palley (2013).

[15] We present a numerical simulation in the Appendix 2 (b).

In Appendix 2 (b), we present one possible value to the equilibrium between the rate of profit and valuation ratio, of this economy. One important issue here is that both total income to capitalists and workers are direct influence from the Current Balance of Payments (difference between exports and imports) and that is a sensitive issue, because in the case of a sustainable deficit in the long-run perspective, both classes will be harmed as we can see in Teixeira and Araújo (1997).

3.3. STABILITY ANALYSIS OF THE MODEL WITH AN OPEN ECONOMY

Using the procedure to the stability from extension without Open Economy, we now have to apply it to this case. From (48) we have:

$$\frac{dr}{dt} = E(r, v_r) = \delta\left[g_n - \frac{s_w}{v}\right] = \delta\left\{g_n + g_n z - \frac{s_w}{v}(1 - t_w) - \left[(1-\beta)\alpha + s_f\left(1 - t_p\right)(1 + z) - s_w(1 - t_w)(1 + z)\right]r + \beta\alpha g_i + (1-\beta)\bar{r} + cg_n v_r - cxg_n\right\}, \delta > 0 \quad (52)$$

Take into account the assumptions (xxv) and (xxvi), but considering the national income from (xxvii), we have:

$$\frac{dv_r}{dt} = E(r, v_r) = \varphi\left[\frac{D_p}{K} - \frac{S_p}{K}\right] = \varphi\left\{\frac{s_w}{v}(1 - t_w) - s_w(1 - t_w)r - s_w(1 - t_w)z\right] - cg_n v_r + cxg_n - xg_n\}, \quad \varphi > 0 \quad (53)$$

To analyse the stability condition, we have to consider the first term of the Taylor expansion and from this obtain the matrix system, as we can see below:

$$\begin{bmatrix} \dot{r} \\ \dot{v}_r \end{bmatrix} = \begin{bmatrix} -\delta\left[(1-\beta)\alpha + s_f\left(1 - t_p\right)(1 + z) - s_w(1 - t_w)(1 + z)\right] & \delta cg_n \\ -\varphi[s_w(1 - t_w) - s_w(1 - t_w)z] & -\varphi cg_n \end{bmatrix} * \begin{bmatrix} r - r_0 \\ v_r - v_{r0} \end{bmatrix} \quad (54)$$

From (54) we have the Jacobian Matrix (J):

$$J[E(r,v_r)] = \begin{bmatrix} -\delta\left[(1-\beta)\alpha + s_f\left(1-t_p\right)(1+z) - s_w(1-t_w)(1+z)\right] & \delta c g_n \\ -\varphi[s_w(1-t_w) - s_w(1-t_w)z] & -\varphi c g_n \end{bmatrix} \quad (55)$$

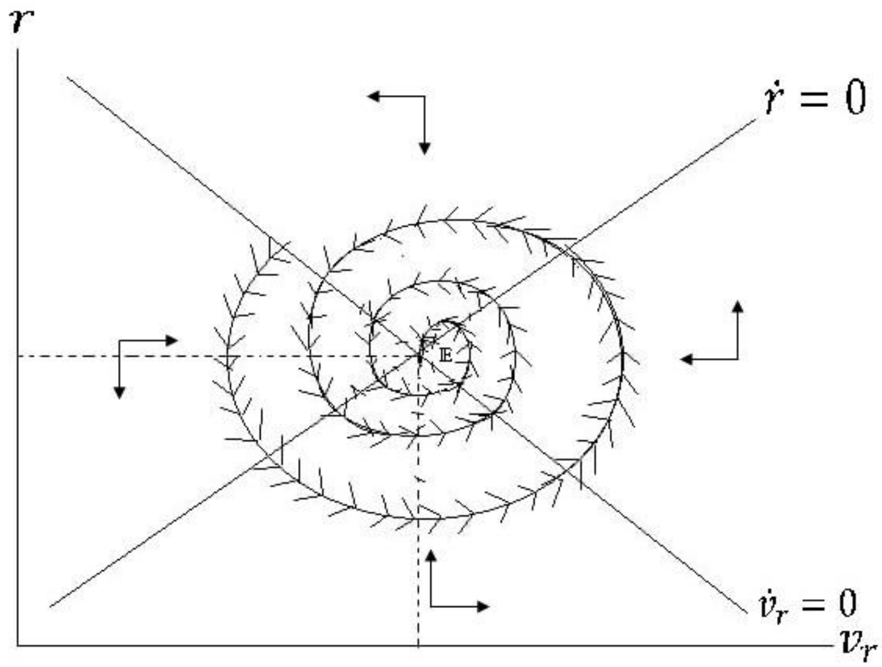
Applying the Olech's Theorem in (54), we have all the tools to analyse the stability in a Matrix 2x2. This is a necessary and sufficient condition if the trace is negative and the determinant is positive, thus:

$$Tr(J) = -\delta\left[(1-\beta)\alpha + s_f\left(1-t_p\right)(1+z) - s_w(1-t_w)(1+z)\right] - \varphi c g_n < 0 \quad (56)$$

$$|J| = \delta\varphi c g_n\left[(1-\beta)\alpha + s_f\left(1-t_p\right)(1+z)\right] > 0 \quad (57)$$

These results show us that with all the assumptions assumed above, the model continue stable as required. From this we conclude that our extension satisfied the stability conditions, *Quod Erat Demonstrandum*.

Figure 2: Dynamic Equilibrium with an Open Economy.



Source: Elaborated by the author.

Equalizing the equation (50) and (51), we determinate the value of the equilibrium β to our extension. Thus:

$$\beta^* = \frac{\left[\frac{s_w}{v}(1-t_w)-x(1-c)g_n\right][\alpha+s_f(1-t_p)(1+z)-s_w(1-t_w)z]-[g_n(1-x)+\alpha\bar{r}+g_n z][cg_n+s_w(1-t_w)]}{(\alpha g_i-\alpha\bar{r})[cg_n+s_w(1-t_w)]+\frac{s_w}{v}(1-t_w)\alpha-x(1-c)g_n\alpha} \quad (58)$$

The expression determinate the β when $r = v_r$, as is presented in the Figure 2. An interesting issue about this stability analysis is that, if we consider a little increase in z, the equilibrium between r and v_r does not modified, but the value of β^* grows. Which means that the government will be forced to increase consumption (incentive to households) more than the profit to maintain the equilibrium condition in long-run.[16] The behaviour of the others parameters in this second extension is the same, as in the extension without an Open Economy.

Note that some care is essential concerning the use of a Political Orientation based on the strategy of export-led growth. It is not difficult to raise the point that such approach necessarily suffers a fallacy of composition. Of course, not all countries can pursue export-led growth simultaneously, unless the domestic economy can expand as a result of a general international expansion of trade. We leave this issue to another opportunity in which we intend of deal with special requirements involving the relevant equations in the present work, income distribution, credit, savings and investments, somewhat take, for convenience, simplified formal specifications from a behavioural perspective.

[16] This result is possible to see in the Appendix 2 by comparing the economy without and with an Open Economy. Considering, as we already show, that the second extension with $z = 0$ is equal to our first extension.

4. CONCLUDING REMARKS

The present dissertation is divided in three parts. The first is the introduction which presents the scope of this work. The second presents a chronological development of the Kaldor-Pasinetti theory before the introduction of the Political Orientation by Charles (2007). In the same part, we consider his hypotheses and correct his mathematical mistakes. We arrive to same conclusion that favouring households will impact negatively to the income distribution. This algebraic manipulation is our first contribution.

In the third section, we presented a new version of the "Kaldor neo-Pasinetti Theorem" with Political Orientation. One relevant contribution is that the government can decide how much will be designated as incentives to profit (firms) and/or to consumption (households). Considering the interval for the consumptions incentives as $0 \leq \beta \leq 1$, our approach is relevant to the post-Keynesian theory since we assume a new way to deal with government decision. Another accomplishment is that we established the difference between increased consumption by government expenditures and the natural consumption of the economy (G_i). Therefore, the impact of profit ratio is positive independently of the governmental choice. This conclusion differs from Charles (2007). His result is more than a "logical sleep". At the subsection 3.1 we show that our model is globally stable. From this we conclude that in the long-run our extension will always converge to equilibrium in steady-state. This new extension can be linked to the neo-Kaleckian theory with respect to the wage-led growth model.

As we have raised in the section 3, it is possible to develop some stimulanting new ideas, showing potential links between Kaldor-Passinetti and neo-Kaleckian views, as suggested by Fonseca and Araujo (2018), since these models have many common principles.

In subsection 3.2, we extend the "Kaldor neo-Pasinetti Model" to the case of an Open Economy, government activities and economy Political Orientation. It shows that globalization can expand the income distribution in the total amount as was showed by Teixeira and Araujo (1997). However, the existence of international financial system increase the valuation ratio, which is positive to the firms growing the value of their bonds by the introduction of foreign equities, one the other hand, decrease the profit rate, however, increase the value of β. Implying that, in this condition, it is necessary that the government grow the incentive to households to maintain the equilibrium. Especially the latter, where we concluded that the profit ratio (with respect to foreign equities) show negative results. Subsection 3.3 deals with our model with an

Open Economy. The long-run term will always converge to the equilibrium in steady-state. We showed that our model can be connected with the export-led growth theory in the post-Kaleckian framework.

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APPENDIX 1: THE CORRECT EQUATION OF INCENTIVE TO CONSUMPTION FOR CHARLES (2007)

Considering the following assumptions:

a) $G_e = \alpha(\overline{C} - C)$;

b) $C = c_w(1 - t_w)W + (1 - s_c)(1 - t_p)P + cG$

Substituting (a) in (b) we have:

c) $C = c_w(1 - t_w)Y - c_w(1 - t_w)P + \alpha(1 - s_c)(1 - t_p)P + \alpha c(v_r - x)I$

Considering the (xiv) and substituting (c) in (a):

$$I + \alpha\overline{C} - \alpha\left[c_w(1 - t_w)Y - c_w(1 - t_w)P + \alpha(1 - s_c)(1 - t_p)P + \alpha c(v_{rw} - x)I\right] = s_c(1 - t_p)P + s_w(1 - t_w)Y - s_w(1 - t_w)P - c(v_{rw} - x)I$$

Dividing the equation per K:

$$g_n + \alpha\overline{c} - \frac{\alpha}{v}(1 - t_w) - \alpha(1 - t_w)r_w + \alpha(1 - s_c)(1 - t_p)P + \alpha c(v_{rw} - x)g_n = s_c(1 - t_p)r_w + \frac{s_w}{v}(1 - t_w) - s_w(1 - t_w)r_w - c(v_{rw} - x)g_n$$

Isolating $\frac{s_w}{v}$:

d) $\frac{s_w}{v} = \frac{1}{(1-t_w)}[g_n + \alpha\bar{c} - \frac{\alpha}{v}(1-t_w) - \propto (1-t_w)r_w + \alpha(1-s_c)\big(1-t_p\big)r +$
$\alpha c(v_{rw} - x)g_n - s_c\big(1-t_p\big)r_w + s_w(1-t_w)r + c(v_{rw} - x)g_n]$

Now considering (9) equal to (d):

$$\frac{1}{(1-t_w)}[xg + c(v_{rw} - x)g_n + s_w(1-t_w)r_w] = \frac{1}{(1-t_w)}[g_n + \alpha\bar{c} - \frac{\alpha}{v}(1-t_w) - \alpha(1-t_w)r_w + \alpha(1-s_c)\big(1-t_p\big)r_w + \alpha c(v_{rw} - x)g_n - s_c\big(1-t_p\big)r_w + s_w(1-t_w)r_w + c(v_{rw} - x)g_n]$$

Isolating r and we have the (3)

$$r = \frac{g_n[1-\alpha c(v_r - x)] + \alpha\bar{c} - \frac{\alpha}{v}(1-t_w)}{\big(1-t_p\big)[\alpha(1-s_c) - s_c] - \propto(1-t_w)}$$

Substituting in (xiv) and manipulating algebraically we can obtain (30):

$$\frac{s_w}{v} = \frac{1}{(1-t_w)}\left(xg_n + c(v_r - x)g_n + s_w(1-t_w)\left\{\frac{g_n[1-\alpha c(v_r - x)] + \alpha\bar{c} - \frac{\alpha}{v}(1-t_w)}{\big(1-t_p\big)[\alpha(1-s_c) - s_c] - \alpha(1-t_w)}\right\}\right)$$

$$\therefore \frac{s_w(1-t_w)}{v} - xg_n + cxg_n = cv_r g_n + s_w(1-t_w)\left\{\frac{g_n[1-\alpha c(v_r - x)] + \alpha\bar{c} - \frac{\alpha}{v}(1-t_w)}{\big(1-t_p\big)[\alpha(1-s_c) - s_c] - \alpha(1-t_w)}\right\}$$

Assuming $\theta = \big(1-t_p\big)[\alpha(1-s_c) - s_c] - \alpha(1-t_w)$

$$\frac{s_w(1-t_w)}{v} - xg_n + cxg_n = cv_r g_n + \frac{s_w(1-t_w)g_n[1-\alpha c(v_r - x)]}{\theta} + \frac{\alpha\bar{c}}{\theta} - \frac{\frac{\alpha}{v}(1-t_w)}{\theta}$$

$$\rightarrow \frac{s_w(1-t_w)}{v} - xg_n + cxg_n = cv_r g_n + \frac{s_w(1-t_w)g_n}{\theta} - \frac{s_w(1-t_w)g_n\alpha c v_r}{\theta} + \frac{s_w(1-t_w)g_n\alpha cx}{\theta} + \frac{\alpha\bar{c}}{\theta} - \frac{\frac{\alpha}{v}(1-t_w)}{\theta}$$

$$\therefore \frac{s_w(1-t_w)}{v} - xg_n + cxg_n - \frac{s_w(1-t_w)g_n}{\theta} - \frac{s_w(1-t_w)g_n\alpha cx}{\theta} - \frac{\alpha\bar{c}}{\theta} - \frac{\frac{\alpha}{v}(1-t_w)}{\theta} = cv_r g_n - \frac{s_w(1-t_w)g_n\alpha c v_r}{\theta}$$

From now, we have to isolate the v_r to obtain (31)

$$\frac{s_w(1-t_w)}{v} - xg_n + cxg_n - \frac{s_w(1-t_w)g_n}{\theta} - \frac{s_w(1-t_w)g_n\alpha cx}{\theta} - \frac{\alpha\bar{c}}{\theta} - \frac{\frac{\alpha}{v}(1-t_w)}{\theta} = v_r\left[cg_n - \frac{s_w(1-t_w)g_n\alpha c}{\theta}\right]$$

$$\rightarrow \frac{s_w(1-t_w)}{v} - xg_n + cxg_n - \frac{s_w(1-t_w)g_n}{\theta} - \frac{s_w(1-t_w)g_n\alpha cx}{\theta} - \frac{\alpha\bar{c}}{\theta} - \frac{\frac{\alpha}{v}(1-t_w)}{\theta} = v_r\left[\frac{cg_n\theta - s_w(1-t_w)g_n\alpha c}{\theta}\right]$$

$$\therefore \frac{\frac{s_w(1-t_w)\theta}{v} - xg_n\theta + cxg_n\theta - s_w(1-t_w)g_n - s_w(1-t_w)g_n\alpha cx - \alpha\bar{c} - \frac{\alpha}{v}(1-t_w)}{cg_n\theta - s_w(1-t_w)g_n\alpha c} = v_r$$

Rearranging the last equation we have the correct valuation ratio:

$$v_r = \frac{xg_n\theta(1-c) + s_w(1-t_w)\left\{\frac{\theta}{v} - g_n + \alpha\left[\frac{(1-t_w)}{v} - \bar{c} - g_n cx\right]\right\}}{cg_n\theta - s_w(1-t_w)g_n\alpha c}$$

APPENDIX 2: NUMERICAL SIMULATIONS

a) Numerical Simulation to the case without an Open Economy.

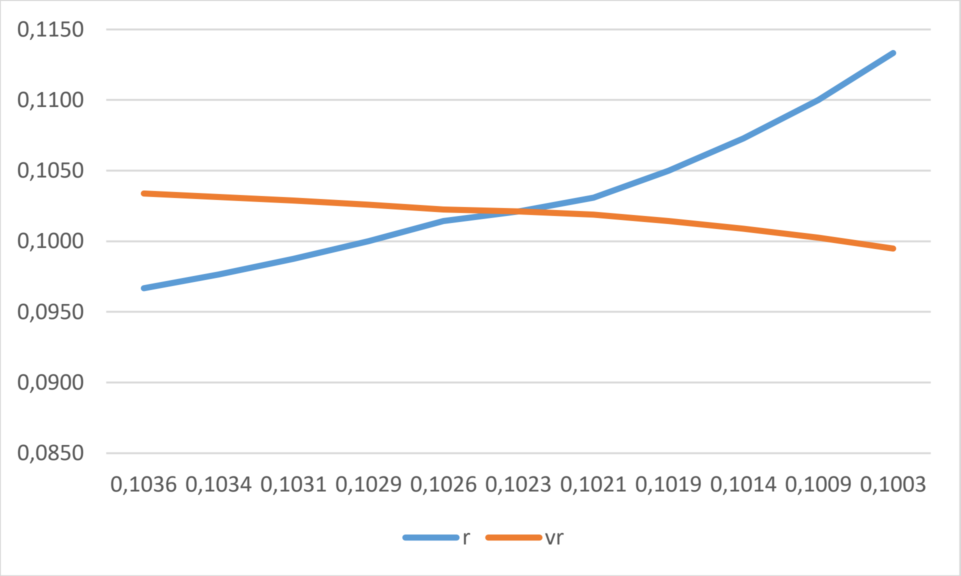
Considering the following assumptions:

$\alpha = 0{,}8$; $s_f = 0{,}9$; $g = 0{,}05$; $t_p = 0{,}20$; $x = 0{,}20$; $t_w = 0{,}30$; $s_w = 0{,}01$; $v = 0{,}9$; $c = 0{,}6$; $K = 50000$; $Y = 10000$; $W = 6000$; $P = 4000$

Chart 1: Numerical Simulation without an Open Economy.

Beta	r	vr	r Kaldor	vr Kaldor
0	0,095789	0,103575	0,044444	0,222222
0,1	0,096667	0,10337		
0,2	0,097647	0,103142		
0,3	0,09875	0,102884		
0,4	0,1	0,102593		
0,5	0,101429	0,102259		
0,542663	0,102102	0,102102		
0,6	0,103077	0,101875		
0,7	0,105	0,101426		
0,8	0,107273	0,100896		
0,9	0,11	0,100259		
1	0,113333	0,099481		

Figure 3: Dynamic Equilibrium without an Open Economy (numerical exercise).



Source: Elaborated by the author.

It is easy to see that this simulation proves all the partial derivatives made in the section 3 and in all cases we have an equilibrium.

b) Numerical Simulation to the case without an Open Economy.

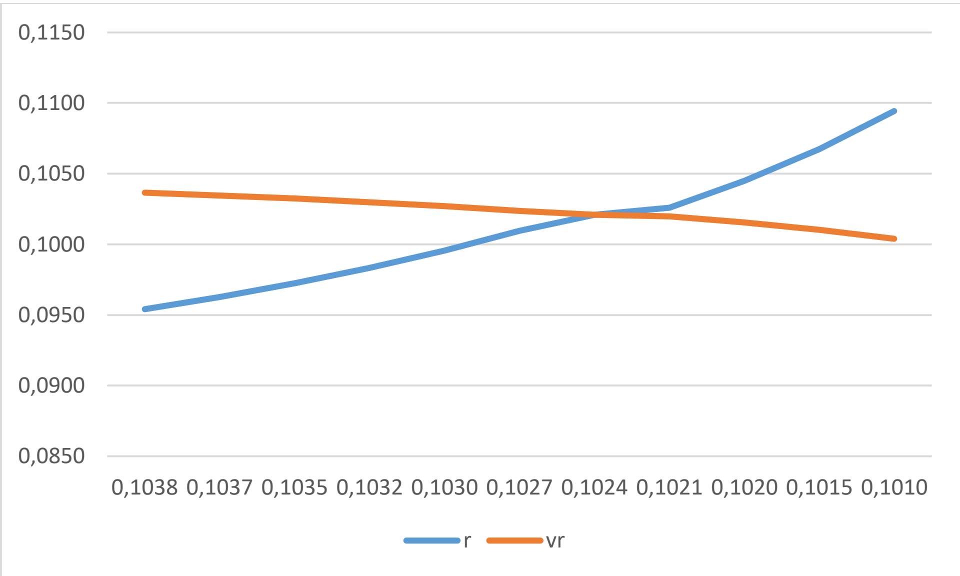
Considering the following assumptions:

$\alpha = 0{,}8$; $s_f = 0{,}9$; $g = 0{,}05$; $t_p = 0{,}20$; $x = 0{,}20$; $t_w = 0{,}30$; $s_w = 0{,}01$; $v = 0{,}9$; $c = 0{,}6$; $K = 50000$; $Y = 10000$; $W = 6000$; $P = 4000$; $z = 0{,}1$

Chart 2: Numerical Simulation with an Open Economy.

Beta	r	vr	r Kaldor	vr Kaldor
0	0,09464	0,103843	0,044444	0,222222
0,1	0,095415	0,103663		
0,2	0,096276	0,103461		
0,3	0,09724	0,103237		
0,4	0,098325	0,102984		
0,5	0,099555	0,102696		
0,6	0,100963	0,102368		
0,671603	0,102102	0,102102		
0,7	0,102589	0,101989		
0,8	0,104489	0,101545		
0,9	0,106737	0,101021		
1	0,10944	0,10039		

Figure 4: Dynamic Equilibrium without an Open Economy (numerical exercise).



Source: Elaborated by the author.

APPENDIX 3: OLECH'S THEOREM [17]

The Olech's Theorem can guarantee a global stability in the plane if the following assumptions hold.

Considering a system with $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in R^2$ and the differential equation is $\dot{x}_n = f_n(x) = \frac{dx_n}{dt}$, $n \in \{1; 2\}$ and in equilibrium $\dot{x}_n = 0$. We can obtain the Jacobian Matrix, as we can see below:

$$J(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix}$$

An equilibrium point is uniformly globally asymptotically stable if respect the following assumptions:

1) The trace of the Jacobian Matrix is negative: $(J) < 0 , \forall\, x \in R^2$.
2) The Jacobian determinant is positive: $|J| > 0 , \forall x \in R^2$.
3) Assuming either: $\frac{\partial f_1}{\partial x_1}\frac{\partial f_2}{\partial x_2} \neq 0$ on R^2 and $\frac{\partial f_1}{\partial x_2}\frac{\partial f_2}{\partial x_1} \neq 0$ on R^2 .

[17] This Appendix is based in Garcia (1972).